

✓✓ Unknown Fact

कलित

ω

Trigonometric
Formula

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$$

$$= \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix} \begin{matrix} [C_1' = C_1 + C_2 + C_3] \\ \\ \end{matrix}$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0$$

$$\sqrt[3]{1} = \chi$$

$$\Rightarrow \chi^3 = 1$$

$$\Rightarrow \chi^3 - 1 = 0$$

$$\Rightarrow (\chi - 1)(\chi^2 + \chi + 1) = 0$$

$$\chi - 1 = 0$$

$$\therefore \chi = 1$$

$$\chi^2 + \chi + 1 = 0$$

$$\chi = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\frac{-1 + \sqrt{-3}}{2} \rightarrow \omega$$

$$\frac{-1 - \sqrt{-3}}{2} \rightarrow \omega^2$$

$$a\tilde{x} + bx + c = 0$$

$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) $1 + \omega + \omega^2$

$$= 1 + \frac{-1 + \sqrt{-3}}{2} + \frac{-1 - \sqrt{-3}}{2}$$

$$= 1 - \frac{1}{2} + \frac{\sqrt{-3}}{2} - \frac{1}{2} - \frac{\sqrt{-3}}{2}$$

$$= 1 - 1 = 0$$

$$\therefore \boxed{1 + \omega + \omega^2 = 0}$$

$$\boxed{\omega^3 = 1}$$

$$\begin{aligned} &\rightarrow 1 + \omega = -\omega^2 \\ &\rightarrow 1 + \omega^2 = -\omega \\ &\rightarrow \omega + \omega^2 = -1 \end{aligned}$$

$$\boxed{\omega^3 = 1}$$

$$\omega^4 = \omega'$$

(ii)

$$\omega^3 = \omega \times \omega^2$$

$$= \left(\frac{-1 + \sqrt{-3}}{2} \right) \left(\frac{-1 - \sqrt{-3}}{2} \right)$$

$$= \left(-\frac{1}{2} \right)^2 - \left(\frac{\sqrt{-3}}{2} \right)^2 = \frac{1}{4} - \left(\frac{-3}{4} \right) = \frac{1}{4} + \frac{3}{4} = 1$$

$$\frac{4}{3} \rightarrow \text{वर्गमूल}$$

$$\begin{vmatrix} \omega^2 & 1 & -\omega \\ 1 & 1 & -\omega \\ -\omega & 1 & -\omega \end{vmatrix} \quad [C_1' = C_1 + C_2 + C_3]$$

$$\Rightarrow \begin{vmatrix} 0 & -\omega - \omega^2 & \omega^2 - 1 \\ 0 & \omega^2 - 1 & 1 + \omega \\ -2\omega & 1 & -\omega \end{vmatrix} \quad \begin{cases} r_1' = r_1 - r_2 \\ r_2' = r_2 - r_3 \end{cases}$$

$$\Rightarrow (-2\omega) \times \begin{vmatrix} -\omega - \omega^2 & \omega^2 - 1 \\ \omega^2 - 1 & 1 + \omega \end{vmatrix}$$

$$\Rightarrow (-2\omega) \times \begin{vmatrix} -\omega(1 + \omega) & \omega^2 - 1 \\ \omega^2 - 1 & 1 + \omega \end{vmatrix}$$

$$\Rightarrow (-2\omega) \times \{-\omega(1 + \omega) - (\omega^2 - 1)\}$$

$$\Rightarrow (-2\omega) \{-\omega(1 + \omega) - (\omega^2 - 1)\}$$

$$\Rightarrow (-2\omega) \{-\omega(1 + \omega) - \omega^2 + 1\}$$

$$\Rightarrow (-2\omega) \{-\omega - \omega^2 - \omega^2 + 1\}$$

$$\Rightarrow -4\omega^3$$

$$\begin{array}{c}
 \cos \beta - \cos \gamma \\
 \frac{\beta - \gamma}{2} \\
 \frac{\beta - \alpha}{2}
 \end{array}
 \left| \begin{array}{c}
 2 \cos \frac{\beta + \gamma}{2} \sin \frac{\beta - \gamma}{2} \\
 2 \sin \frac{\beta + \gamma}{2} \sin \frac{\gamma - \beta}{2}
 \end{array} \right| \rightarrow \left| \begin{array}{c}
 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \\
 2 \sin \frac{\alpha + \beta}{2} \sin \left\{ -\left(\frac{\alpha - \beta}{2} \right) \right\}
 \end{array} \right|
 \begin{array}{c}
 2 \cos \frac{\beta + \gamma}{2} \\
 2 \sin \frac{\beta + \gamma}{2}
 \end{array}$$

$$\begin{array}{c}
 2 \frac{\alpha - \beta}{2} \times 2 \sin \frac{\beta - \gamma}{2} \\
 \cos \frac{\alpha + \beta}{2} \quad \cos \frac{\beta + \gamma}{2} \\
 -\sin \frac{\alpha + \beta}{2} \quad -\sin \frac{\beta + \gamma}{2}
 \end{array}
 \Rightarrow \left| \begin{array}{c}
 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \\
 -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}
 \end{array} \right|
 \begin{array}{c}
 2 \cos \frac{\beta + \gamma}{2} \\
 -2 \sin \frac{\beta + \gamma}{2}
 \end{array}$$

$$\left\{ \sin \frac{\alpha - \beta}{2} \sin \frac{\beta - \gamma}{2} \right\} - \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\beta + \gamma}{2} \right) + \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\beta + \gamma}{2} \right) \left\{$$