



BNP

Topic List

*

নির্ণায়কের *Math solve (Part – 03)*

*

Hard problem

$$\begin{vmatrix} x^3 & y^3 & z^3 \\ 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} (xyz - 1)$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ x-y & y-z & z-x \\ x^2-y^2 & y^2-z^2 & z^2-x^2 \end{vmatrix} (xyz - 1) \begin{bmatrix} c'_1 = c_1 - c_2 \\ c'_2 = c_2 - c_3 \end{bmatrix}$$

$$= - \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix} [r_2 \Rightarrow r_3]$$

$$= + \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} [r_1 \Rightarrow r_2]$$

$$= (xyz - 1) \times \begin{vmatrix} x-y & y-z & z-x \\ (x-y)(x+y) & (y-z)(y+z) & (z-x)(z+x) \end{vmatrix}$$

$$= (xyz - 1) (x-y)(y-z)(z-x)$$

$$\begin{vmatrix} 1+a^2-b^2 & \boxed{2ab} & \boxed{-2b} \\ \boxed{2ab} & 1-a^2+b^2 & \boxed{2a} \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

$$= \begin{vmatrix} 1+\check{a}-\check{b}-(-2\check{b}) & 0 & -2\check{b} \\ 0 & 1-\check{a}+\check{b}+2\check{a} & 2\check{a} \\ 2\check{b}-(\check{b}-\check{a}\check{b}-\check{b}) & -2\check{a}+\check{a}-\check{a}^3-\check{a}\check{b} & 1-\check{a}-\check{b} \end{vmatrix} \left[\begin{array}{l} c_2' = c_2 + a c_3 \\ c_1' = c_1 - b c_3 \end{array} \right]$$

$$= \begin{vmatrix} 1+\check{a}+\check{b} & 0 & -2\check{b} \\ 0 & 1+\check{a}+\check{b} & 2\check{a} \\ \check{b}(1+\check{a}+\check{b}) & -\check{a}(1+\check{a}+\check{b}) & 1-\check{a}-\check{b} \end{vmatrix}$$

$$= (1+\check{a}+\check{b})^2 \begin{vmatrix} 1 & 0 & -2\check{b} \\ 0 & 1 & 2\check{a} \\ \check{b} & -\check{a} & 1-\check{a}-\check{b} \end{vmatrix}$$

2072 10370
1010 1002

$$= (1+\check{a}+\check{b})^2 \begin{vmatrix} 1 & 2\check{a} \\ -\check{a} & 1-\check{a}-\check{b} \end{vmatrix} \\ = (1+\check{a}+\check{b})^2 (1-\check{a}-\check{b}) \\ = (1+\check{a}+\check{b})^2 (1+\check{a})$$

$$\begin{vmatrix}
 0 & x^2 & y^2 \\
 (z+x+y)(z+x-y) & y^2 & (x+y)^2 \\
 (z+x+y)(z-x-y) & (x+y)^2 & x^2
 \end{vmatrix}$$

$$\begin{aligned}
 & (z+x+y)(z+x-y) - (y^2 - x^2)(z-x-y) \\
 & + 2xyz + 2x^2y - 2xy^2 \\
 & - y^2z + x^2y + y^3 \\
 & - x^2z - x^3 - x^2y \\
 & yz
 \end{aligned}$$

$$\begin{aligned}
 &= (x+y+z)^2 \begin{vmatrix} y+z-x & 0 & x^2 \\ -2z & z+x-y & y^2-x^2 \\ 0 & z-x-y & (x+y)^2 \end{vmatrix} \left[R_2' = R_2 - R_1 \right] \\
 &= (x+y+z)^2 \left[(y+z-x) \begin{vmatrix} z+x-y & y^2-x^2 \\ z-x-y & (x+y)^2 \end{vmatrix} + x^2 \begin{vmatrix} -2z & 0 \end{vmatrix} \right] \\
 &= (x+y+z)^2 \left[(y+z-x) \left\{ (x+y)(z+x-y) - (z-x-y)(y^2-x^2) \right\} \right. \\
 & \quad \left. - 2x^2z \right]
 \end{aligned}$$

$$= \frac{1}{abc} \begin{vmatrix} a\vec{b} + a\vec{c} & a\vec{b} & a\vec{c} \\ a\vec{b} & b\vec{c} + a\vec{b} & b\vec{c} \\ c\vec{a} & b\vec{c} & a\vec{c} + b\vec{c} \end{vmatrix}$$

$$= \frac{1}{abc} \times \cancel{abc} \begin{vmatrix} \vec{b} + \vec{c} & \vec{a} & \vec{a} \\ \vec{b} & \vec{c} + \vec{a} & \vec{b} \\ \vec{c} & \vec{c} & \vec{a} + \vec{b} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2\vec{c} & -2\vec{b} \\ \vec{b} - \vec{c} & \vec{a} & -\vec{a} \\ \vec{c} & \vec{c} & \vec{a} + \vec{b} \end{vmatrix}$$

$$\begin{vmatrix} \vec{b} - \vec{c} & \vec{a} & \vec{a} \\ \vec{c} & \vec{c} + \vec{a} & \vec{b} \\ -\vec{a} & \vec{c} & \vec{a} + \vec{b} \end{vmatrix}$$

$$= \frac{1}{\vec{b} \cdot \vec{c}} \begin{vmatrix} 0 & -2\vec{b} \cdot \vec{c} & -2\vec{b} \cdot \vec{c} \\ \vec{b} \cdot \vec{c} & \vec{a} \cdot \vec{b} & -\vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{c} & \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} & \end{vmatrix}$$

$$= \frac{1}{\vec{b} \cdot \vec{c}} \begin{vmatrix} 0 & -2\vec{b} \cdot \vec{c} & -2\vec{b} \cdot \vec{c} \\ \vec{b} \cdot \vec{c} & \vec{a} \cdot \vec{b} & -\vec{a} \cdot \vec{c} \\ 0 & 0 & -2\vec{b} \cdot \vec{c} \end{vmatrix}$$



$$\begin{bmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ a^4 - 2a & b^4 - 2b & c^4 - 2c \end{bmatrix} \text{ হলে প্রমাণ কর যে, } \det(A) = abc(abc - 2) (a-b)(b-c)(c-a).$$

